SBE A-LEVEL MATHEMATICS BRIDGING COURSE BOOKLET

All students wishing to do A-Level Mathematics must complete the booklet and hand it in by 1st September. You must show enough working out to prove that you have not used a calculator for the non-calculator questions. All the material in the booklet is GCSE standard which you must be familiar with to be successful when studying A-Level Mathematics.

FORMULA THAT YOU WILL NEED TO LEARN

Area of a triangle = 0.5 x base x height

Area of a circle = πr^2

Circumference of a circle = $2\pi r$

Volume of a cylinder = $\pi r^2 h$

Volume of a prism
= area of cross-section x length

Quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sine rule $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$

Area of a triangle = 0.5abSinC

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FRACTIONS

A-Level maths deals with fractions rather than decimals as they are easier to manipulate.

Key points:
Convert between mixed and top heavy
Add and subtract
Multiply and divide

Mixed fractions to top-heavy

$$3\frac{4}{5} = \frac{(3\times 5)+4}{5} = \frac{19}{5}$$

Top-heavy to mixed fractions

$$\frac{43}{7} = 6\frac{1}{7}$$

$$43 \div 7 = 6r1$$

QUESTIONS

Write these mixed fractions as top-heavy fractions

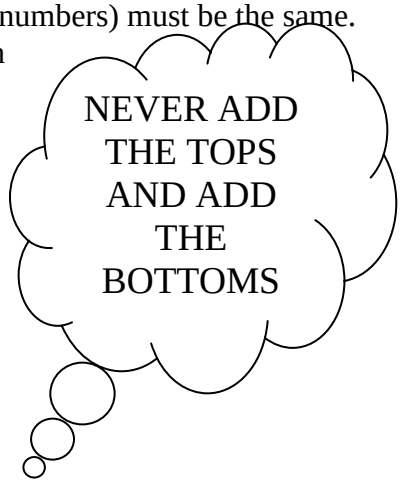
1. $2\frac{4}{5}$ 2. $3\frac{1}{4}$ 3. $1\frac{11}{13}$

Write these hop-heavy fractions as mixed fractions

4. $\frac{21}{5}$ 5. $\frac{81}{4}$ 6. $\frac{62}{7}$

Adding and subtracting fractions

Before you can add or subtract any fraction, their denominators (bottom numbers) must be the same. Try to find the LCM of the denominators – if you can't the multiply each denominator by the other.
Turn any mixed fraction into a top-heavy one will help.



$$\underset{x2}{\frac{5}{6}}-\underset{x3}{\frac{1}{4}}=\frac{10}{12}-\frac{3}{12}=\frac{10-3}{12}=\frac{7}{12}$$

$$3\frac{5}{8}+1\frac{2}{3}=\frac{29}{\underset{x3}{8}}+\frac{5}{\underset{x8}{3}}=\frac{87}{24}+\frac{40}{24}=\frac{127}{24}=5\frac{7}{24}$$

Algebraic fractions are done in the same way. Multiply out the brackets in the numerator but not in the denominator.

$$\frac{4}{(x+1)}+\frac{3}{(x-2)}=\frac{4(x-2)}{(x+1)(x-2)}+\frac{3(x+1)}{(x-2)(x+1)}$$

$$=\frac{4(x-2)+3(x+1)}{(x+1)(x-2)}=\frac{4x-8+3x+3}{(x+1)(x-2)}=\frac{7x-5}{(x+1)(x-2)}$$

QUESTIONS

Work these out

7. $1\frac{3}{5}+2\frac{8}{15}$

8. $5\frac{1}{2}-2\frac{3}{4}$

9. $\frac{7}{10}+\frac{2}{5}+\frac{3}{4}$

10. $\frac{x-1}{3}+\frac{x+2}{4}$

11. $\frac{x+2}{2}-\frac{2x+1}{7}$

Multiplying fractions

- Write the fractions in top-heavy form first, cancelling if possible.
- Multiply the numerators together, then multiply the denominators together.

$$\frac{4}{5}\times\frac{6}{7}=\frac{4\times6}{5\times7}=\frac{24}{35}$$

$$1\frac{2}{5}\times3\frac{1}{8}=\frac{7}{5}\times\frac{25}{8}=\frac{7\times25}{5\times8}=\frac{175}{40}=\frac{35}{8}$$

$$\frac{2a}{3b}\times\frac{5ab}{4c^2}=\frac{10a^2b}{12bc^2}=\frac{5a^2}{6c^2}$$

Dividing fractions

- Write the fractions in top-heavy form first, cancelling if possible.
- Flip the second fraction upside down and change the ÷ to a x.

$$\frac{7}{10}\div\frac{3}{5}=\frac{7}{10}\times\frac{5}{3}=\frac{7\times5}{10\times3}=\frac{35}{30}=\frac{7}{6}=1\frac{1}{6}$$

$$4\frac{3}{5}\div2\frac{3}{4}=\frac{23}{5}\div\frac{11}{4}=\frac{23}{5}\times\frac{4}{11}=\frac{23\times4}{5\times11}=\frac{92}{55}=1\frac{37}{55}$$

QUESTIONS

Work these out

12. $3\frac{2}{3}\times2\frac{1}{2}$

13. $3\frac{1}{2}\div2\frac{3}{5}$

14. $3\times\frac{4}{5}$

15. $6\div\frac{3}{4}$

INDICES

Key points:
Laws of indices
Simplification
Convert between index notation and symbolic notation

LAWS OF INDICES – you need to know these well. At A-Level you will constantly need to use these laws.

$a^m \times a^n = a^{m+n}$ $\quad a^b \div a^c = a^{b-c}$ $\quad (a^p)^q = a^{pq}$

$b^0 = 1$ $\quad a^{\frac{1}{n}} = \sqrt[n]{a}$ $\quad a^{\frac{m}{n}} = \sqrt[n]{a^m}$

$x^{-m} = \frac{1}{x^m}$ $\quad x^{\frac{-1}{n}} = \frac{1}{\sqrt[n]{x}}$ $\quad x^{\frac{-m}{n}} = \frac{1}{\sqrt[n]{x^m}}$

"-" sign means one over, not making the number negative.

These laws only work when the base numbers are the same.
They would not work for $2^7 \times 3^5$ as 2 and 3 are not the same value.

Simplify:

$2^7 \times 2^3$
$2^7 \times 2^3 = 2^{7+3} = 2^{10}$

$7^6 \div 7$
$7^6 \div 7 = 7^6 \div 7^1 = 7^{6-1} = 7^5$

Write in index notation:

$\frac{1}{\sqrt{5}}$
$5^{\frac{-1}{2}}$

$\sqrt[4]{7^8}$
$\sqrt[4]{7^8} = 7^{\frac{8}{4}}$

Write in symbolic form:

$11^{\frac{1}{2}}$
$11^{\frac{1}{2}} = \sqrt{11}$

$x^{\frac{-3}{5}}$
$x^{\frac{-3}{5}} = \frac{1}{\sqrt[5]{x^3}}$

Evaluate

$64^{\frac{1}{3}}$
$64^{\frac{1}{3}} = \sqrt[3]{64} = 4$

$8^{\frac{-2}{3}}$
$8^{\frac{-2}{3}} = \sqrt[3]{8^2} = 2^2 = 4$

QUESTIONS

Write in index form:

16. $2^3 \times 2$
17. $4^5 \div 4^{-2}$
18. $(7^2)^3$
19. $\frac{1}{2^3}$
20. $\sqrt{8}$
21. $3^{-3} \div 3^4$
22. $\frac{1}{x^2}$
23. $a^4 \times a^7$
24. $(m^3)^5$
25. $z^{0.5} \times z^{1.2}$

Write in symbolic form:

26. $7^{\frac{1}{3}}$
27. 25^{-2}
28. $y^{\frac{4}{5}}$
29. a^{-1}
30. 3^{-3}
31. $x^{\frac{-2}{5}}$
32. $\left(9^{\frac{1}{2}}\right)^{-2}$
33. $y^{\frac{1}{8}}$
34. $4x^{-2}$
35. $(5x)^{\frac{-1}{2}}$

Evaluate.

36. $4^{0.5}$
37. 54^0
38. 13^1
39. $27^{\frac{1}{3}}$
40. $64^{\frac{-2}{3}}$
41. 10^2
42. $32^{\frac{1}{5}}$
43. $\left(\frac{125}{8}\right)^{\frac{1}{3}}$
44. $\left(\frac{25}{9}\right)^{\frac{-3}{2}}$
45. $\left(\frac{169}{144}\right)^{\frac{1}{2}}$

SURDS

Key points:
Simplification
Rationalising
$\sqrt{6}\times\sqrt{6}=\sqrt{36}=6$

Simplify $\sqrt{50}$ First write out the factor pairs of 50: 1, 50 2, 25
Then choose the pair that contains the largest square number: 2, 25
Re-write 50 as 25 x 2

$$\sqrt{50}=\sqrt{25\times 2}=\sqrt{25}\times\sqrt{2}=5\times\sqrt{2}=5\sqrt{2}$$

Rationalising the denominator means to get rid of the surds in the denominator, without changing the value of the original fraction.

Rationalise $\frac{4}{\sqrt{5}}$

$$\frac{4}{\sqrt{5}}=\frac{4}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{4\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}}=\frac{4\sqrt{5}}{5}$$

Multiply numerator and denominator by $\sqrt{5}$.
(The surd in the denominator)

Rationalise $\frac{7}{2-\sqrt{3}}$

Multiply the denominator by the conjugate of the denominator.

(the same function that is in the denominator, but with the Opposite sign)

$$\frac{7}{2-\sqrt{3}}=\frac{7}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{7\times\left(2+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}=\frac{14+7\sqrt{3}}{4-3}=14+7\sqrt{3}$$

Remember: $(2-\sqrt{3})(2+\sqrt{3})$ $= 2x2 + 2x\sqrt{3} - 2x\sqrt{3} - \sqrt{3}x\sqrt{3}$
$= 4 + 2\sqrt{3} - 2\sqrt{3} - 3 = 4 - 3 = 1$

QUESTIONS

Simplify the following:

46. $2\sqrt{27}$ 47. $\sqrt{75}-\sqrt{48}$

Rationalise:

48. $\frac{3+\sqrt{2}}{\sqrt{8}}$ 49. $\frac{4}{2+\sqrt{3}}$ 50. $\frac{3-\sqrt{5}}{2-\sqrt{7}}$

FACTORISING (NON-QUADRATICS)

Key points:
Put into brackets
Common factors (letter or number or both in every term)

$7x^2 + 21xy = 7x(x + 3y)$

Check your answers by multiplying back out

$6y^2x^3 - 9y^3x^2 = 3y^2x^2(2x - 3y)$

$3x(x + 2) - 4(x + 2) = (x + 2)(3x - 4)$

Brackets can also be common to both terms.

QUESTIONS

Factorise the following:

51. $3ab^3 - 3a^3b$

52. $6 + 2x^2$

53. $4x(2x + 3) - 3(2x + 3)$

54. $5x^2(x - 1) - 2x(x - 1)$

TRANSPOSING FORMULA

Key points:
Make x the subject
Use inverse operations

A = B + Mx

In this equation, A is "the subject" in order to make "x" the subject, the equation needs to be rearranged.

-B -B
A = B + Mx
÷M ÷M
A – B = Mx
$\frac{A-B}{M} = x$

Whatever you do to one side of the equation, you must do to the other

x(y – a) = e — make (y) the subject

x(y – a) = e — multiply out brackets first
+xa +xa
xy – xa = e — take all terms not containing "y" to the other side from y.
÷x ÷x
xy = e + xa — divide by x to leave y by itself (the subject)
$y = \frac{e+xa}{x}$

QUESTIONS

Make the letter in brackets the subject of the equation:

55. $\frac{ra-t}{S} = v$ (a)

56. $\frac{f(Nb-e)}{m} = A$ (b)

57. $\frac{y+x}{y-x} = 3$ (y)

58. $\sqrt[a]{\frac{x^2-n}{m}} = \frac{a^2}{b}$ (x)

QUADRATICS

Key points:
Factorising
Complete the square
Solving
Quadratic formula

Quadratic equations or expression are those which have an x^2 in them. There are no higher powers such as x^3. Some will also have an x term and a constant term, some will have one of these terms missing.

Factorising

either one or two brackets, depending on the terms you have

$4x^2 - 6x = 2x(2x - 3)$ common factors

$x^2 - 25 = (x - 5)(x + 5)$ difference of two squares

$x^2 - 7x + 10 = (x - 5)(x - 2)$ factors of the number at the end that add to give the middle term.

$2x^2 + 7x + 6$ multiply together the coefficients of the first and last terms

$2 \times 6 = 12$

List the factor pairs of this product

1	12
2	6
3	4

Choose the pair that add to give the middle term.

$= 2x^2 + 4x + 3x + 6$ Re-write this middle term as the sum of the factor pair

$= 2x(x + 2) + 3(x + 2)$ Factorise each half of the expression separately to give the same bracket (x+2)

$= (x + 2)(2x + 3)$ Final factorisation is the common bracket (x + 2) and then the remaining terms in the second bracket (2x + 3)

Completing the square

$x^2 + 5x - 6$ Halve the coefficient of x to go in the back part of the bracket.

Divide the coefficient of x by 2

$5 \div 2 = 5/2$

$= (x + 5/2)^2 - (25/4) - 5$ Subtract the square of the number in the back of the bracket

Square the (5/2) and subtract $(5/2)^2 = (25/4)$

$= (x + 5/2)^2 - 45/4$ Simplify to get your final answer

Solving:

you know you have to solve because there will be an equals sign.
First move all the terms to one side so that your equation equals zero.

- by factorisation

$4x^2 - 6x = 0$
$2x(2x - 3) = 0$
$2x = 0$ or $2x - 3 = 0$
$X = 0$ or $2x = 3$
$x = 3/2$

$x^2 - 25 = 0$
$(x - 5)(x + 5) = 0$
$x - 5 = 0$ or $x + 5 = 0$
$x = 5$ or $x = -5$

$x^2 - 7x + 10 = 0$
$(x - 5)(x - 2) = 0$
$x - 5 = 0$ or $x - 2 = 0$
$x = 5$ or $x = 2$

$2x^2 + 7x + 6 = 0$
$2x^2 + 4x + 3x + 6 = 0$
$2x(x + 2) + 3(x + 2) = 0$
$(x + 2)(2x + 3)$
$x + 2 = 0$ or $2x + 3 = 0$
$x = -2$ or $x = -3/2$

Put each of the brackets equal to zero and then make x the subject for each

- by completing the square

$x^2 + 5x - 6 = 0$
$(x + 5/2)^2 - (25/4) - 6 = 0$
$(x + 5/2)^2 - 49/4 = 0$
$(x + 5/2)^2 = 49/4$

$$\left(x + \frac{5}{2}\right)^2 = \frac{49}{4}$$

$$\left(x + \frac{5}{2}\right) = \sqrt{\frac{49}{4}}$$

$$\left(x + \frac{5}{2}\right) = \pm\frac{7}{2}$$

$$x = \pm\frac{7}{2} - \frac{5}{2}$$

$$x = \frac{7}{2} - \frac{5}{2} \text{ or } -\frac{7}{2} - \frac{5}{2}$$

$$x = 1 \text{ or } -3$$

When you square root, remember to do $\pm$ the answer to get both solutions

- by using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

for $ax^2 + bx + c$

$3x^2 - 2x - 4 = 0$
a = 3, b = -2, c = 4

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

substitute the values for a, b, c into the equation and solve.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-4)}}{2(3)}$$

$$x = \frac{2 \pm \sqrt{4 + 48}}{6}$$

$$x = \frac{2 \pm \sqrt{52}}{6}$$

$$x = \frac{1 \pm \sqrt{13}}{3}$$

Either leave in surd form or type this into your calculator to get a decimal answer.

QUESTIONS

Factorise:
59. $x^2 - 9$ 60. $x^2 + 3x + 2$ 61. $3x^2 - 16x + 21$

Complete the square:
62. $x^2 - 4x$ 63. $x^2 + x + 3$ 64. $x^2 + 3x$

Solve by factorising:
65. $x^2 - 9 = 0$ 66. $x^2 + 3x + 2 = 0$ 67. $2x^2 + 9x + 9 = 0$ 68. $5x^2 + 13x + 6 = 0$
69. $3x^2 - 16x + 21 = 0$

Solve by completing the square
70. $x^2 - 4x = -2$ 71. $x^2 + x + 3 = 0$ 72. $x^2 + 3x = 0$ 73. $2x^2 + 8x - 5 = 0$

Solve by using the formula – leave in surd form
74. $2x^2 + 11x + 5 = 8$ 75. $3x^2 - 2x - 5 = 0$ 76. $x^2 - 7x + 2 = 0$
77. $3 + 4x - 2x^2 = 0$

INEQUALITIES

Key points:
Solving
Unknowns on both sides
Involving fractions

Treat inequalities like equations. NEVER MULTIPLY OR DIVIDE, always add or subtract

+1 +1
$2x - 1 > 5$
÷2 ÷2
$2x > 5$

$x > 5/2$

always have x positive

$2(x - 3) < 14$ expand brackets first
+6 +6
$2x - 6 < 14$
÷2 ÷2
$2x < 20$

$x < 10$

QUESTIONS

Solve:

78. $4w - 5 \leq 2w + 2$

79. $2(5x - 1) \leq 2x + 3$

80. $3(2x + 1) < 15$

81. $\frac{x+6}{4} \geq 1$

82. $\frac{2x}{5} > 3$

83. $\frac{4x+3}{2} \geq 11$

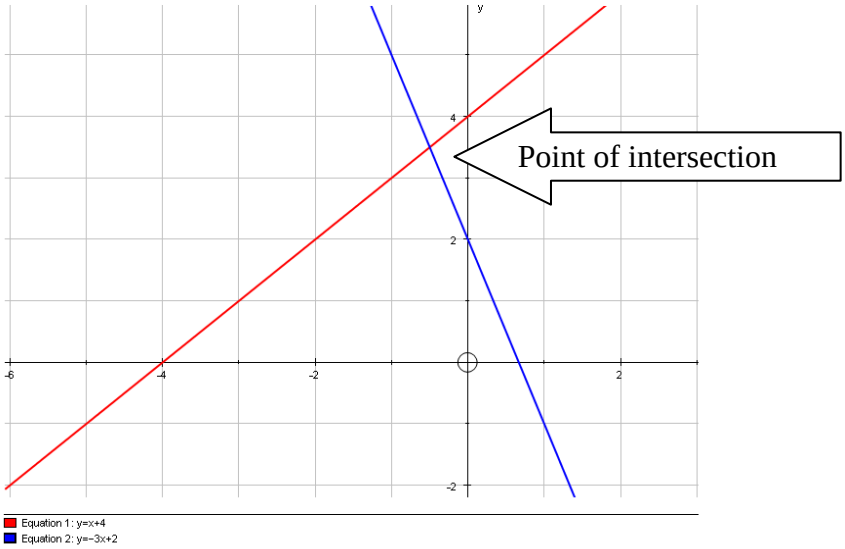
84. $\frac{5x-2}{4} \leq -1$

SIMULTANEOUS EQUATIONS

Key points:
Graphically
By substitution
One linear and one quadratic

Graphically - To solve simultaneous equations graphically, draw both the graphs and write down the coordinates of where they intersect.

Solve: $y = x + 4$
$y = -3x + 2$



By substitution

Solve $x - 3y = 7$ (i)
$x + y = 3$ (ii)

Rearrange one of the equations to make x or y the subject the substitute into the other equation.

Rearranging (i) gives $x = 7 + 3y$ (iii)
Sub into (ii) $x + y = 3$
$7 + 3y + y = 3$
$4y = -4$
$y = -1$

Now you have found y, don't forget to find x!

Find x by substituting the y value into (i) or (ii)
Sub y into (i) $x - 3y = 7$
$x - 3(-1) = 7$
$x + 3 = 7$
$x = 4$

One linear one quadratic

Solve $y + 2x = 9$ (i)
$y^2 + x^2 = 18$ (ii)

Which equation looks the simplest? Why?

Rearrange the simplest equation.
Rearranging (i) gives $y = 9 - 2x$

Substitute this into (ii)

$y^2 + x^2 = 18$
$(9 - 2x)^2 + x^2 = 18$
$81 - 36x + 4x^2 + x^2 = 18$
$5x^2 - 36x + 81 = 18$
$5x^2 - 36x + 81 - 18 = 0$
$5x^2 - 36x + 63 = 0$

Remember:
$(9 - 2x)^2 = (9 - 2x)(9 - 2x)$
$= 81 - 18x - 18x + 4x^2$
$= 81 - 36x + 4x^2$

now solve

$(5x - 21)(x - 3) = 0$
$x = 21/5$ or $x = 3$

Now find BOTH the y values

When x = 3: $y = 9 - 2x$
$y = 9 - 2(3) = 3$

when x = 21/5: $y = 9 - 2x$
$y = 9 - 2(21/5) = 3/5$

Final answer: (3, 3) and (21/5, 3/5)

QUESTIONS

85. $y = x^2 - 2x$
$y = x + 4$

86. $y = 7x - 8$
$y = x^2 - x + 7$

87. $y = x^2 - 3x + 7$
$5x - y = 8$

88. $x^2 + y^2 = 20$
$y = x - 2$

STRAIGHT LINE GRAPHS

Key points:
Sketching graphs
Gradient
Parallel and perpendicular

Sketching

You only need two points to sketch a straight line – where it crosses the x-axis and where it crosses the y-axis.

Sketch $y = 4x + 8$

Where it crosses the y – axis (where x = 0)	$y = 4(0) + 8 = 8$	(0, 8)
Where it crosses the x – axis (where y = 0)	$0 = 4x + 8$	
	$4x = 8$ so $x = 2$	(2, 0)

Plot these two points and join them with a straight line.
To sketch a curve you need to know two things: the general shape and where it crosses both the axes.

Sketch $y = x^2 + 3x + 2$

Where it crosses the y – axis (where x = 0)	$y = 0^2 + 3(0) + 2 = 2$	(0, 2)
Where it crosses the x – axis (where y = 0)	$0 = x^2 + 3x + 2$	
	$0 = (x + 2)(x + 1)$	
	$x + 2 = 0$ or $x + 1 = 0$	
	$x = -2$ or $x = -1$	(-1, 0) and (-2, 0)

Plot these points and draw a positive quadratic curve through the points.

Gradient

To find the gradient of a straight line, rearrange the equation into the form $y = mx + c$. The gradient will be the coefficient (number infront of) of x.

Find the gradient of:

$y = -2x + 4$ gradient is -2

$y = \frac{4}{5}x - \frac{1}{2}$ gradient is 4/5

$3y = 2x - 1$ this needs to be re-arranged first

$y = \frac{2}{3}x - \frac{1}{3}$ gradient is 2/3

$6x + 2y - 7 = 0$
$2y = -6x + 7$

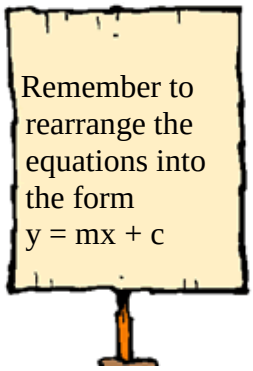
$y = -3x + \frac{7}{2}$ gradient is -3

Perpendicular and parallel lines

Two lines are parallel if their gradients are the same.
Two lines are perpendicular if the product of their gradients is -1. ($m_1 \times m_2 = -1$)

Which of the following are parallel or perpendicular?

A: $y = -x + 7$
B: $4x + 12y = 5$
C: $y = x - 3$
D: $5x + y = 12$
E: $y = 2x + 4$
F: $2y = x - 7$
G: $x + 3y = 10$
H: $y = 5 - 5x$
I: $y = 3 - 2x$



QUESTIONS

Find the gradient of the following lines

89. $5y - 7x - 2 = 0$
90. $11x - 2y = -1$
91. $0.5y + 4x = 16$
92. $-3x + 2y - 12 = 4$

93. State which of the following are parallel or perpendicular, giving a full explanation for your choice.

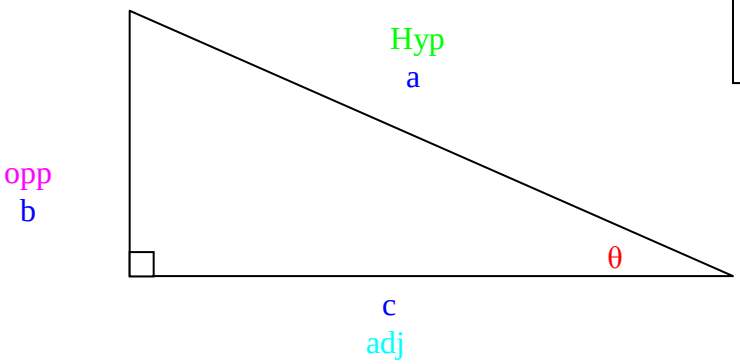
A: $y = 3x - 7$
B: $y = 4x + 2$
C: $3y - 12x + 18 = 0$
D: $2y + 12 = 7x$
E: $4y + x = 2$
F: $3y + 6x = 5$

TRIGONOMETRY AND PYTHAGORAS

Key points:
Pythagoras
SOHCAHTOA

PYTHAGORAS

$a^2 = b^2 + c^2$



These are used only in right angle triangles

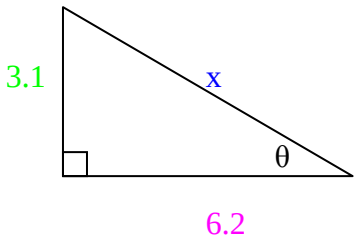
TRIGONOMETRY

$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$

$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$

Find the missing length and angle in this triangle.



Use Pythagoras to find x

$x^2 = 3.1^2 + 6.2^2$
$x^2 = 48.05$
$x = \sqrt{48.05} = 6.93$

Use Trigonometry to find θ

$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$

$\tan\theta = \frac{3.1}{6.2}$

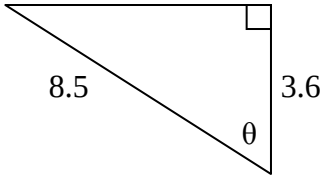
$\theta = \tan^{-1}\frac{3.1}{6.2}$

$\theta = 26.6^\circ$

QUESTIONS

Find the missing sides and angles in the following triangles:

94.



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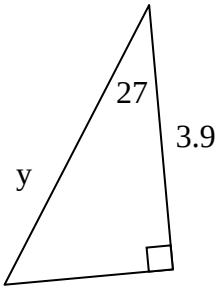

96.